Using asymmetric Okun law and Phillips curve for potential output estimates: an empirical study for Romania

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Abstract: Having in view the economic crisis that Romania has crossed and the delayed effects on the macroeconomic indicators, the objective of the paper is the study of the modifications incurred on the unemployment time varying NAIRU and output gaps paths.

Using some computed data in previous works, and also quarterly data from NIS Statistics and NBR statistics, in the paper it is estimated time varying NAIRU using Phillips asymmetric curve approach and Kalman Filter approach as well. The output gaps computing is based on time varying NAIRU recalculated in this article and symmetric and asymmetric dynamic Okun’s laws.

Keywords: potential GDP, Kalman filter, Phillips curve, Okun’s law, HP filter.

JEL: C13, E01, E19.

Introduction

The deep global recession between 2008-2009, that for Romania has had delayed effects until the end of 2010 presents evident characteristics of persistence on actual and potential output in many countries. The persistency of the effects is debt to the reducing of capital accumulation that further has been completed with the reduction of the economy’s potential output (Ball, 2014).

In many countries output is still highly depressed in 2014, and there is little recovery in the next five years (Reinhart and Rogoff (2014), IMF forecasting (2009)).

These delayed effects of the crisis were appointed “hysteresis” (Blanchard and Summers, 1986).

In one of the papers, Summers (2014) shows: “This financial crisis has confirmed the doctrine of hysteresis more strongly than anyone might have supposed”.

In the same paper, analysing the actual state of the US economy, he refers to “secular stagnation” characterized by the reduction of the potential level and growth rates, negative natural interest rates that imply the ineffectiveness of monetary policies in raising the growth rates of GDP and potential GDP.

These characteristics of “secular stagnation” have been identified for USA and Japan, but some of the features can also be found in the countries of Europe

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post crisis, such as lower productivity and growth rate of actual and potential output, slow rate of recovery as well.

These phenomena have significant impact on the labour market, on time varying NAIRU, with high costs on the sides of quality of life.

The objective of this empirical work is to study the impact of the economic crisis in Romania on time varying NAIRU and output gaps.

The paper is structured in five chapters that include introduction, estimating NAIRU using asymmetric Phillips curve and Kalman filter approach, structural approach of the output gaps, empirical application for Romania, conclusions and further topics.

1. Estimating time varying NAIRU using asymmetric Phillips curve and Kalman filter approach

1.1 Estimating time varying NAIRU using asymmetric Phillips curve

Regarding the Phillips curve, there are different opinions with respect of its sharp. Some of the researchers considering that Phillips curve have convex sharp (Turner, 1995, Debelle and Laxton, 1997), others consider concave sharp (Eisner, 1997 and Stiglitz, 1997), and finally same considering linear Phillips curve (Gordon, 1984).

The difference of opinion on the empirical evidence may imply high costs regarding economical results of political decisions.

The Phillips curve is asymmetric because, in general, unemployment below NAIRU tends to result in increasing and eventually explosive inflation, whereas excess unemployment will have a diminishing effect on inflation.

Debelle and Laxton (1997), Laxton ed al. (1999) suggest an analytic form for the asymmetric Phillips curve:

$$\pi_t = \lambda \pi_t^e + \gamma(u_t^N - u_t)\phi_t + \epsilon_t$$

(1)

Where: $u_t^N$ is unemployment time varying NAIRU, $\pi_t^e$ is the expected inflation and $\phi_t$ is defined as the lower bound on $u$, reflecting short-run constraints on the level of unemployment:

$$\phi_t = \text{MAX}(0, u_t^N - \bar{u})$$

(2)

For their empirical study, Laxton ed al (1999) used the value $\bar{u} = 4$.

The time varying NAIRU could be computed using asymmetric Phillips curve, using the algorithm presented below.

First it is estimated the asymmetric Phillips curve, writing it in a linear form:

$$\pi_t = \lambda \pi_t^e + \gamma \bar{U}_t + \epsilon_t$$

(3)

With $\bar{U}_t = (u_t^N - u_t)/\phi_t$. 

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The data series for $\tilde{U}_t$ could be computed using previous estimations of unemployment NAIRU ($u^N_t$) and considering for $\phi_t$ computing, a value for $\overline{u}$ equal with fixed Ball-Mankiw NAIRU.

In (3), $\pi_t^c$ is the expected inflation that is assumed to follow an adaptive mechanism.

The errors $\varepsilon_t$ are computed taking the differences:

$$\pi_t - \lambda \pi_t^c - \gamma \tilde{U}_t = \varepsilon_t$$

The time varying NAIRU, using the asymmetric Phillips curve will be:

$$u^N_t = (\pi_t - \lambda \pi_t^c - \varepsilon_t \left( \frac{u_t - \phi_t}{\gamma} \right)) + u_t$$

### 1.2 Time varying NAIRU estimates using Kalman filter approach

In order to build the Kalman system (Andrei et al (2014)) for determining the unobservable time varying NAIRU it is assumed a generalised Phillips curve with $\Delta \pi_t$, the actual inflation first order difference, $\theta u_t$ actual unemployment first order difference ($\Delta u_t$) delay and an AR(2) process for the unemployment NAYRU dynamics:

$$\Delta \pi_t = \alpha_0 \Delta \pi_{t-1} + \alpha_1 \Delta \pi_{t-2} - \theta (u_t - u^N_t) - \gamma_1 \Delta u_t - \gamma_2 \Delta u_{t-1} + \varepsilon_t$$

$$u^N_t = \beta_1 u_{t-1} + \beta_2 u_{t-2} + \eta_t$$

In (6) and (7) respectively, $\varepsilon_t$ and $\eta_t$ are random processes with the zero mean and covariance $\text{Cov}(\varepsilon_t) = R$, $\text{Cov}(\eta_t) = Q$, respectively.

The Kalman form of the dynamic model is:

$$[X_{1,t} = \beta_1 X_{1,t-1} + \beta_2 X_{2,t-1} + \eta_{1,t}]
X_{2,t} = X_{1,t-1}
Y_t = \theta X_{1,t} + \varepsilon_t$$

The first and second equations are the state equations, while the third equation in the state form of the dynamic system (8) is known as the observational or output equation.

The output, observable variable $Y_t$ is defined as:

$$Y_t = \Delta \pi_t - \alpha_0 \Delta \pi_{t-1} - \alpha_1 \Delta \pi_{t-2} + \theta u_t + \gamma_1 \Delta u_t + \gamma_2 \Delta u_{t-1}$$

and the state, unobservable vector:

$$X_{1,t} = u^*_t, X_{2,t} = u^*_{t-1}.$$
The output variable data series (9) is completely determined using the statistical data of price inflation and actual unemployment.

The application of Kalman filter results in approximation of the unobserved variables

\[
\begin{bmatrix}
X_{1,t} \\
X_{2,t}
\end{bmatrix} = \begin{bmatrix}
u_t^{NAIRUKF} \\
u_{t-1}^{NAIRUKF}
\end{bmatrix}
\]
to be computed (11)

2. Structural approach of the Output Gaps

Dynamic Symmetric and Asymmetric Okun’s law

Okun’s law is a macroeconomic relationship between output and unemployment, hence relating the level of activity in the goods market to the level of activity in the labour market over the business cycle.

The empirical estimates of Okun’s coefficient, which is a measure of the responsiveness of unemployment to output growth, are important insofar as they indicate the cost of unemployment in terms of output.

The meaning of asymmetry is that the response of unemployment to output growth is different when the economy is expanding from that when the economy contracts.

This is different from the conventional specification, which encompasses symmetry in the sense that expansions and contractions in output have the same absolute effect on unemployment.

Okun’s (1962) first equation captures the relationship between the changes in the unemployment rate and the growth in real output:

\[
\Delta u_t = \alpha + \beta \Delta y_t + \varepsilon_t
\] (12)

with \(u_t\) the unemployment rate, \(y_t\) natural log of real GDP, \(\Delta\) first difference operator.

The parameter \(\beta\) (which is called “Okun’s coefficient”) is negative, and the ratio: \((-\alpha / \beta\) gives the growth rate of output consistent with a stable unemployment rate, or how quickly the economy would need to grow to maintain a given level of unemployment.

Okun’s second equation is the following:

\[
u_t = \gamma + \Delta y_t + \varepsilon_t
\] (13)

where \(\Delta y_t = y_t - y_t^{POT}\) is output gap.

Many economists now use a dynamic version of Okun’s law (Owyang, Sekhposyan 2012) given by:

\[
\Delta u_t = a_1 + b_1 \Delta y_t + b_2 \Delta y_{t-1} + b_3 \Delta y_{t-2} + c_1 \Delta u_{t-1} + c_2 \Delta u_{t-2} + \varepsilon_t
\] (14)
This dynamic version of Okun’s law bears some similarities to the original difference version of Okun’s law, but it captures actual, and two periods lag correlation between changes in unemployment rate and real output growth. Andrei et al. (2009), estimated a variant of Weber (1995) dynamic Okun’s Law:
\[ u_t - u_t^{NAWRU} = a \times \text{output}_t \times \text{gap}_t + b \times u_{t-1}^{NAWRU} + c \times u_{t-2}^{NAWRU} + \varepsilon_t. \] (15)
and the results confirmed that economic mechanisms of reaction do function normally in Romania and that unemployment is sensitive to changes in economy. The results were pertinent in both econometric and economic points of view.

Silvapulle et al. (2004) used symmetric model and the data for United States for the post war period. They concluded that the short-run effects of positive cyclical output on cyclical unemployment are quantitatively different from those of negative ones, and the data are consistent with the assertion that cyclical unemployment is more sensitive to negative than to positive cyclical output. Their symmetric model is:
\[ u_t = \sum_{j=1}^{p} \alpha_j u_{t-j} + \sum_{j=1}^{q} \beta_j y_{t-j} + \varepsilon_t \] (16)

With \( u_t \), cyclic unemployment, \( u_t^N \), with \( u_t \) unemployment rate, \( u_t^N \) unemployment NAIRU rate. \( y_t^c \) is cyclical output in logarithms, \( y_t^c = y_t - y_t^{POT} \), with \( y_t \) real GDP in logarithms, \( y_t^{POT} \) potential output in logarithms. The long-run multiplier, \( \phi \) giving the effect of output on unemployment is given by:
\[ \phi = \left\{ \sum_{j=1}^{p} \beta_j \right\} \left\{ 1 - \sum_{j=1}^{q} \alpha_j \right\}^{-1} \] (17)

which is Okun’s coefficient and it is expected to be negative.

The output gap is then determined as:
\[ u_t - u_t^N = \phi(y_t - y_t^{POT}) \] (18)

With \( u_t^N \) computed using the two methods mentioned.

The second version of the model takes into account the conclusions of the previous application of the symmetric variant that leads to underestimates of unemployment rate increases in contractions and overestimates of decreases in the unemployment rate during expansions. Therefore, the model is transformed decomposing the variable \( y_t^c \) in two components corresponding to the phase of the economic cycle.

The asymmetric model therefore is:
\[ u_t^c = \sum_{j=1}^{p} \alpha_j u_{t-j}^c + \sum_{j=1}^{q} \theta_j y_{t-j}^c + \eta_j y_{t-j}^c - \varepsilon_t \] (19)
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Where \( y_t^+ = y_t^e \delta_t \) with \( \delta_t \) a dummy variable that is equal to 1 when \( y_t^e > 0 \) (during expansions) and 0 when \( y_t^e < 0 \) (in depressions), and \( y_t^- = y_t^e \gamma_t \) with \( \gamma_{t-j} \) also a dummy that takes value 1 when \( y_t^e < 0 \) and 0 when \( y_t^e > 0 \).

The asymmetric Okun coefficients in this case are:

\[
\phi^+ = \left( \sum_{j=1}^{q} \theta_j / (1 - \sum_{j=1}^{p} \alpha_j) \right) \text{ for recessions}
\]

\[
\phi^- = \left( \sum_{j=1}^{q} \eta_j / (1 - \sum_{j=1}^{p} \alpha_j) \right) \text{ for expansions}
\]

and are expected to be negative.

The asymmetric output gaps are therefore:

\[
u_t - u_t^{\text{NAIRU}} = \phi^+ (y_t - y_t^{\text{POT}}) \quad \text{for output gap in expansions (that is } u_t - u_t^{\text{NAIRU}} < 0) \]

and \( u_t - u_t^{\text{NAIRU}} = \phi^- (y_t - y_t^{\text{POT}}) \) for output gap in recessions (that is \( u_t - u_t^{\text{NAIRU}} > 0 \)).

Recall the fact that:

- \( \text{output gap} > 0 \) is the inflationary gap (aggregate demand > aggregate supply) unemployment gap < 0, inflation gap > 0
- \( \text{output gap} < 0 \) is the recessionary gap that could lead to deflation, unemployment gap > 0, inflation gap < 0.

3. Empirical application for Romania

3.1 Computing time varying NAIRU using Kalman filter approach and asymmetric Phillips curve method

The two unobservable variables (NAIRU and potential GDP) are calculated usually one using the other, and these can be determined only approximatively. That is why the estimation of NAIRU unemployment rate in two steps is more than justified in order to increase the accuracy of the results. The first step is based on the previous determinations of that indicator (Andrei, Păun, 2014) and are used in this paper as input data for parameter estimates.

The coefficients of the asymmetric Phillips curve, are estimated using the linear form (3) with \( \hat{U}_t = (u_t^N - u_t) / (u_t - \phi) \) completely determined with actual unemployment rate (NIS statistics data), \( u_t^N \) previous approximation and \( \pi = 7.546 \)
considered equal with fixed NAIRU computed for Romania using Ball=Mankiw method (Andrei, Păun, 2014), so \( \phi_i = \text{MAX}(0, u_i^N - 7.546) \).

The equation to be estimated is:
\[
\pi_t = \lambda \pi_t^e + \gamma (u_t^N - u_t) / (u_t - \phi_t) + \varepsilon_t^\pi
\]
(22)

And the estimated function obtained is:
\[
\pi_t = 0.139159 \pi_t^e - 0.097538 \bar{U}_t + \varepsilon_t
\]
(23)

The errors are computed using (4), taking the differences:
\[
\varepsilon_t = -0.139159 \pi_t^e + 0.097538 \bar{U}_t
\]
(24)

The time varying NAIRU is then computed using (5) and previous results:
\[
u_t^N = (\pi_t - 0.139159 \pi_t^e - \varepsilon_t) \left( \frac{u_t - \phi_t}{0.097538} \right) + u_t
\]
(25)

![Figure 1. Asymmetric Phillips curve NAIRU (series 1) and actual unemployment rate (series 2)](image)

The results are satisfactory both in the econometric sense and economic sense.

In economic sense, the unemployment gaps marks emphasized the recessionary gaps between Q36-Q43 that is 2009.1-2010.4, the period of the recent economic crisis in Romania.

Also it can be seen a high volatility of the unemployment actual rate after the quarter 28-29, that means before and after the year of entrance of Romania in EU, taking in account the delay caused by hysteresis phenomena.

The negative gaps after 2010.4 caused by the hysteresis phenomena, follows a period between Q25-Q36 characterised by positive unemployment gaps, in which aggregate demand is higher than aggregate supply.

The time varying Kalman filter NAIRU is computed according to the algorithm presented in Andrei et al. (2014).
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The computed variable NAIRU approximate is used then to estimate the parameters of the Phillips curve (6) and of the AR (2) process given by (7), the series: $\Delta \pi_t$, $\Delta \pi_{t-1}$, $\Delta \pi_{t-2}$, $(u_t - u_t^{\text{NAIRUPC}})$, $\Delta u_t$, $\Delta u_{t-1}$, are known (NIS statistics).

The Kalman filter method was then applied to determine second step variable Kalman filter NAIRU $u_t^{\text{NAIRUKF}}$.

The resulted can be seen in the figure below.

![Figure 2. Kalman Filter NAIRU (series 1) and actual (series 2) unemployment rates](image)

Kalman filter NAIRU maintains approximatively the same tendency of the actual unemployment rate. The unemployment deviations are smaller than those obtained with asymmetric Phillips curve NAIRU, but comply with the economy’s cycles.

### 3.2 Computing Output Gaps using symmetric and asymmetric dynamic Okun law

The previous use of Okun law in our works for computing the output gaps was in Andrei et al. 2009 and Andrei Păun 2011. Now we have resumed the estimation of dynamic symmetric and asymmetric Okun law for Romania, taking into account the evolution of the phenomena and considering the quarterly data from 2000.1-2014.2.

It was used Akaike information criterion and Schwarz Bayesian information criterion (STATA, version 12) in order to determine the order of lags and it was obtained: $p = 2$ for unemployment cyclical component and $q = 3$ for output cyclical component.
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In order to estimate the symmetric dynamic Okun, it was used the series of $u_t^c$ and $y_t^c$, constructed using HP filter cyclical component of unemployment actual rate and of the logarithm of actual real GDP.

The estimated symmetric dynamic Okun law is:

$$u_t^c = 0.09872u_{t-1}^c + 0.00893u_{t-2}^c - 0.226133y_{t-1}^c - 0.0260y_{t-2}^c - 0.006143y_{t-3}^c + \epsilon,$$

Using (17), the computed Okun coefficient is:

$$\phi = \frac{\beta_1 + \beta_3 + \beta_4}{1 - \alpha_1 - \alpha_2} = -0.2894 \quad \phi = -0.2894$$

that means that the increase by one percentage point of the output gap in logarithms implies the reduction on unemployment gap of $-0.2894$ percentage points.

The asymmetric Okun Law considering the same order of delays is:

$$u_t^c = 0.08526u_{t-1}^c + 0.009759u_{t-2}^c - 0.2983y_{t-1}^c - 0.0844y_{t-2}^c - 0.009219y_{t-3}^c + 0.09527y_{t-1}^c +$$

$$+ 0.01537y_{t-2}^c + 0.00329y_{t-3}^c + \epsilon$$

$$\phi^- = \frac{\theta_1 + \theta_2 + \theta_3}{1 - \alpha_1 - \alpha_2} = -0.4441, \quad \phi^+ = \frac{\eta_1 + \eta_2 + \eta_3}{1 - \alpha_1 - \alpha_2} = -0.1259$$

The results reveal clearly the asymmetric influence of unemployment to output growth in the expansions and contractions: the increase of one percentage point of the output gap implies a reduction of $-0.1259$ percentage points of unemployment gap and a decrease of one percentage point of the output gap in logarithms imply an increase of the unemployment gap of $0.4441$ percentage points.

Using Okun coefficients and the unemployment gaps computed with both Kalman filter approach and asymmetric Phillips curve approach, the approximations of the output gaps with both symmetric and asymmetric dynamic Okun law were computed.

**Figure 3. Output gaps using KFNAIRU and symmetric Okun coefficient**
It can be seen that the gaps reflect the cyclic periods crossed by the Romanian economy, especially the last economic crisis, that, under that estimation, was reduced from 0.97% in q31 to -3.89% in q33 and to -8.9% in q37.

![Figure 4. Output gaps with KFNAIRU and asymmetric Okun coefficients](image)

The output gaps computed with KFNAIRU and asymmetric Okun coefficients lead to some more prominent decreases in the crisis: from 1.9% in q30, to -11.34% in q35 and -10.87% in q38, and, around the q41 becomes positives, but with lower values of actual and potential GDP.

![Figure 5. Output gaps with asymmetric Phillips curve NAIRU and symmetric Okun coefficient](image)

The higher amplitude of the gaps in the period of q9-q21 is due to the variations of NAIRU computed on the basis of asymmetric Phillips curve. The gaps have higher amplitude in expansions than in recessions. During q35-q39 the gaps were -2.6%, -3.01%, -3.9%, -3.02% and -2.2%.
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Figure 6. Output gaps with asymmetric Phillips curve NAIRU and asymmetric Okun coefficient

The output gaps computed with asymmetric NAIRU Phillips curve and symmetric and asymmetric dynamic Okun coefficients make more evident the expansions and the depressions, the gaps being more emphasized than the case of KF NAIRU.

The gaps in depressions with the two time varying NAIRU estimates and the asymmetric dynamic Okun coefficients are higher than with the symmetric dynamic Okun coefficient.

4. Conclusions and Further Topics

The present study started from the assumption that the strong decrease in GDP and high unemployment rates in relative short period of crises in Romania, and also the high volatility of GDP data before and after Romania’s accession to the European Union are phenomena that require special attention.

The variable NAIRU was computed, using two methods: Kalman filter approach and asymmetric Phillips curve approach and quarterly data from 2001q1 to 2014q2.

The computed unemployment gaps resulted, follow the Romanian’s cyclical pattern during the period considered and are approximatively consistent as signs but are much different from the point of view of the amplitude of the gaps.

The symmetric and asymmetric Okun coefficients were determined using the two variant of the Silvapulle et al (2004) Okun’s law.

The first symmetric curve was estimated in two variants using the KFNAIRU and asymmetric Phillips curve NAIRU and the HP Filter cyclical components for unemployment actual rate and for real GDP in logarithms.

The second asymmetric curve was estimated also in two variants using the same data as the symmetric one, but dividing this time the data of HP filter cyclical component of real GDP in logarithms in two series, one of them with positive values and one with negative ones.
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Taking into account the large scale of methods used for approximations of unobserved indicators like NAIRU and potential GDP or output gaps, the fact that they can be determined only approximatively and taking into account the importance accorded by European Commission to this indicators for fiscal and structural deficits surveillance, some authors propose methods to mix the computed results of different methods in order to obtain more accurate results. Our future concerns will be to continue the application of different methods for computing NAIRU, NAWRU and output gaps and to mix the results obtained for Romania.

References


